

Research Article

The New Calculus

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Newton's calculus revised.

Logical Derivatives

1. $\frac{d}{dx}(\sin x) = \cos x + \Delta x^2 * \sin x$
2. $\frac{d}{dx}(\cos x) = -\Delta x^2 * \cos x - \sin x$
3. $\frac{d}{dx}(\tan x) = \frac{-(\cos 2x - \sin 2x)}{\cos^2 x}$
4. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2} - \Delta x^2 \cdot x}$
5. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2} + \Delta x^2 \cdot x}$
6. Power Rule $\frac{d}{dx}(x^n) = \frac{n! \cdot x^{n-k} \cdot \Delta x^{k-1}}{k! \cdot (n-k)!}$

7. Irrational Functions

$$\sum_{p=0}^{p=m} \frac{n! * y^{m-p} * \Delta y^{p-1}}{p! \cdot (m-p)!} * \frac{d(y)}{dx} = \sum_{k=0}^{k=n} \frac{n! \cdot x^{n-k} \cdot \Delta x^{k-1}}{k! \cdot (n-k)!}$$

8. The Natural Logarithm Function $\frac{d}{dx}(\ln x) = \ln\left(\frac{x+1}{x}\right)$

9. $\frac{d}{dx}\left(\frac{1}{x}\right) = \left(\frac{-1}{x*(x+1)}\right)$

Proof Of Newton's Rule

The Power Rule establishes derivatives for expressions with a higher power. An elementary proof of this rule was popularized in early England of 16th century by Issacc Newton.

Let me assume a curve of n-degree of the following simplified type with no y-intercepts.

$$y(x)=x^n$$

Differentiating this curve using Newton's elementary difference quotient, I write-

$$\frac{d}{dx}(x^n) = \lim_{\Delta x \rightarrow 0} \left[\frac{(x+\Delta x)^n - x^n}{\Delta x} \right]$$

Newton's binomial expansion consisting of binomial coefficients is read 'n choose k' - (n,k)

Increasing my mathematical reasoning, I see further expansion results; thus,

$$\frac{d(x^n)}{dx} = \lim_{\Delta x \rightarrow 0} \left[\sum_{k=0}^{k=n} (n, k) \cdot x^{n-k} \cdot \Delta x^{k-1} - x^n \cdot \Delta x^{-1} \right]$$

Extracting the first term gives me simplifies Newton's further efforts. And

$$\frac{d(x^n)}{dx} = \lim_{\Delta x \rightarrow 0} (n, k) * x^n * \Delta x^0 + \lim_{\Delta x \rightarrow 0} \left[\sum_{k=0}^{k=n} x^{n-k} \cdot \Delta x^{k-1} - x^n \cdot \Delta x^{-1} \right]$$

The expression simplifies. Regardless of all our mistakes, I go on and all of the above equals

$$\frac{d(x^n)}{dx} = x^{-1} * x^n + (n, k) * \lim_{\Delta x \rightarrow 0} \left[\sum_{k=1}^{k=n} x^{n-k} \cdot \Delta x^{k-1} - x^n \cdot \Delta x^{-1} \right]$$

First and last terms cancel; thus,

$$\frac{d(x^n)}{dx} = (n, k) * \lim_{\Delta x \rightarrow 0} \left[\sum_{k=1}^{k=n} x^{n-k} \cdot \Delta x^{k-1} \right]$$

Lacks logic and requires additional and daring analysis.

For k=1 and $\lim_{\Delta x \rightarrow 0} [\Delta x^{-1}] = 0$, Newton has strenuously proven that

$$\frac{d(x^n)}{dx} = n * x^{n-1}$$

Thus, the advent and spearhead of Newton's integer-based calculus of non-linear curves with infinitesimal measurements requirements fuels science and mathematics for the next five hundred years.

Mathematical Induction.

Let η be a positive integer. It is required that -

$$\frac{d(r^\eta)}{dx} = \eta r^{\eta-1}$$

Obviously, as η =1, then substitution grants us-

$$\frac{d(r^1)}{dx} = (1) \cdot r^{1-1}$$

Because

$$r^k \cdot \frac{d}{dx}(r) + r \cdot \frac{d}{dx}(r^k) = r^k + k \cdot r \cdot r^{k-1} = r^k + k \cdot r^k$$

Which equals

$$r^k \cdot (k + 1) = (k + 1) \cdot r^{(k+1)-1}$$

And true for all positive integers greater or at most equal to m=1.

Because the k+1 derivative of the n-th degree curve is expressed as

$$\frac{d^{k+1}}{dx^{k+1}} (x^n) = \frac{d^k}{dx^k} \left[\frac{d}{dx} (x^n) \right] = n * \frac{d^k}{dx^k} [(x^{n-1})]$$

which equals,

$$= n * \frac{d^{k-1}}{dx^{k-1}} \left[\frac{d}{dx} (x^{n-1}) \right] = n * \frac{d^{k-1}}{dx^{k-1}} \left[\frac{d}{dx} \left(\frac{x^n}{x} \right) \right]$$

and,

$$n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{n * x * x^{n-1} - x^n}{x^2} \right) \right] = n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{n * x^n - x^n}{x^2} \right) \right]$$

Which again equals,

$$= n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{n * x^n - x^n}{x^2} \right) \right] = n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{(n-1)x^n}{x^2} \right) \right]$$

$$= n * \frac{d^{k-1}}{dx^{k-1}} [(n-1)x^{n-2}] = n * (n-1) \frac{d^{k-1}}{dx^{k-1}} [(x^{n-2})]$$

And generally, after k=n-th successive derivatives of x^n, the above results in

$$\frac{d^{k+1}}{dx^{k+1}} (x^n) = \frac{d^{k-n}}{dx^{k-n}} \left[\frac{d}{dx} (x^n) \right] = n! \frac{d^0}{dx^0} [(x^{n-k})] = n! ; n = k, x = 0;$$

However, additional analysis requires increasing our rationale of Newton's logical deductions. I call this new logic- "The Logical Derivative" which increases the precision of Newton's overall results. Further analysis and assuming a convergent value of less than 1 of the decrements due to a geometric expansion with differences in the computed derivatives grants enough justification to call this new set of rules- The New Calculus.

Now, let F(x,Δx) represent the derivative of (x+Δx)^n; that is -

$$F(x, \Delta x) = \frac{d(x+\Delta x)^n}{dx}$$

; a Taylor series representation of the first derivative of x^n -

$$\sum_{k=0}^{k=\infty} \frac{a_k}{k!} * X^{n-k}$$

And Taylor series coefficients -

$$a_k(0) = \frac{f^k(0)}{k!}$$

Let

$$u = (x + \Delta x)$$

and

$$u^n = (x + \Delta x)^n$$

Factoring Δx^n from inside the parenthesis gives me-

$$u^n = \Delta x^n * (1 + x * \Delta x^{-1})^n = \Delta x^n * (1 + x)^n$$

Differentiating left and right sides gives me-

$$\frac{d(u)^n}{dx} = \frac{d(u)^n}{dx} * \frac{du}{dx} = \Delta x^n * \frac{d(1+x)^n}{dx} = n * \Delta x^n * (1 + x)^{n-1}$$

Setting $x = 0$ gives me-

$$\frac{d}{dx} (x + \Delta x)^n = n * \Delta x^n$$

Because the $k + 1$ derivative of the n -th degree curve is expressed as

$$\frac{d^{k+1}}{dx^{k+1}} ((x + \Delta x)^n) = \frac{d^k}{dx^k} \left[\frac{d}{dx} ((x + \Delta x)^n) \right] = n * \frac{d^k}{dx^k} [(x + \Delta x)^{n-1}]$$

which equals,

$$= n * \frac{d^{k-1}}{dx^{k-1}} \left[\frac{d}{dx} (x + \Delta x)^{n-1} \right] = n * \frac{d^{k-1}}{dx^{k-1}} \left[\frac{d}{dx} \left(\frac{(x+\Delta x)^n}{(x+\Delta x)^1} \right) \right]$$

and,

$$n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{n * (\Delta x + x) * (x + \Delta x)^{n-1} - (x + \Delta x)^n * (1)}{(x + \Delta x)^2} \right) \right] = n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{n * (x + \Delta x)^n - (x + \Delta x)^n}{(x + \Delta x)^2} \right) \right]$$

which again equals,

$$\begin{aligned} &= n * \frac{d^{k-1}}{dx^{k-1}} \left[\left(\frac{(n-1) * (x + \Delta x)^n}{(x + \Delta x)^2} \right) \right] = n * \frac{d^{k-1}}{dx^{k-1}} [(n-1) * (x + \Delta x)^{n-2}] \\ &= n * (n-1) * \frac{d^{k-1}}{dx^{k-1}} [((x + \Delta x)^{n-2})] \end{aligned}$$

And generally, after $k=n$ -th successive derivatives of $(x + \Delta x)^n$, the above results in

$$\begin{aligned} \frac{d^{k+1}}{dx^{k+1}} ((x + \Delta x)^n) &= \frac{d^{k-n}}{dx^{k-n}} \left[\frac{d}{dx} (x + \Delta x)^n \right] = n! \frac{d^0}{dx^0} [(x + \Delta x)^{n-k}] = n! * \Delta x^k ; n = \\ k \frac{d^k}{dx^k} (x + \Delta x)^k &= n * (n-1) * (n-2) * \dots \dots \Delta x^k = n! * \Delta x^k \end{aligned}$$

The expression on the right side is proportional to the k -th derivative- it's variance.

Δx was differentiated zero times and thus must be proportional $k =$

n . The Taylor series representation of the derivative of the n degree curve is therefore,

$$\frac{d(x+\Delta x)^n}{dx} = \sum_{k=0}^{k=\infty} \frac{n! * x^{n-k} * \Delta x^{k-1}}{k! * (n-k)!}; \text{ linear and directly proportional.}$$

A logical extension of newton's calculus.

Convergence Of Inverse Decrement

Let $y(x)=1/x$ denote an inverse curve and x_1 and x_2 denote two different points on the curve, thus, a decrement of the inverse function can be expressed as-

$$\Delta y = \frac{1}{x_2} - \frac{1}{x_1} = -\left(\frac{x_2 - x_1}{x_2 * x_1}\right)$$

Denoting $\Delta x = x_2 - x_1$

Substituting into the quotient results in-

$$\Delta y = \frac{1}{x_2} - \frac{1}{x_1} = -\left(\frac{\Delta x}{x_2 * x_1}\right)$$

Dividing through with Δx gives me the secant line of the inverse curve; and that gives me,

$$\frac{\Delta y}{\Delta x} = \frac{1}{x_2} - \frac{1}{x_1} = -\left(\frac{\Delta x * \Delta x^{-1}}{x_2 * x_1}\right)$$

If I allow Δx to approach infinitesimally and negligent values as x_2 approaches x_1 , I get, a limit to derivative and-

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -\left(\frac{\lim_{\Delta x \rightarrow 0} (\Delta x^{-1})}{x_2 * x_1}\right)$$

Therefore,

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -\left(\frac{\lim_{\Delta x \rightarrow 0} (\Delta x^{-1})}{x_2 * x_1}\right)$$

Dividing through with Δx gives me;

$$\frac{dy}{dx} = -\left(\frac{\lim_{\Delta x \rightarrow 0} (\Delta x^{-1})}{x_1 * x_1}\right)$$

$$\frac{dy}{dx} = -\left(\frac{\lim_{\Delta x \rightarrow 0} (\Delta x^{-1})}{x_1 * x_1}\right)$$

Subtracting, I get

$$\frac{dy}{dx} + \left(\frac{\lim_{\Delta x \rightarrow 0} (\Delta x^{-1})}{x_1 * x_1}\right) = 0$$

Newton's derivative of $1/x$ is and equals

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

Substituting, I get

$$-\frac{1}{x^2} (1 - \Delta x^{-1}) = 0$$

$\Delta x^{-1} = 1$; zero factor property.

Numerical Value of Newton's Decrement

Thus, to compute the approximate value of Newton's decrement, I, first, recall that the

$$\sin \Delta x = \Delta x; \Delta x \sim 0;$$

Thus, I can assume a correlation between Δx and $\sin \Delta x$.

For decimal values near the origin, I can use Newton's method to estimate the x-intercept between $\sin \Delta x$ and Δx . That is, $\sin \Delta x - \Delta x = 0$; thus,

$$\Delta X_{n+1} = \Delta X_n - \frac{f(\Delta x)}{f'(\Delta x)}$$

$$\Delta X_{n+1} = \Delta X_n - \frac{\sin \Delta x - \Delta x}{\cos \Delta x + \Delta x * \sin \Delta x}$$

$$\Delta X_n = .05; \sin \Delta x - \Delta x = .049979 - .05;$$

$$\cos .05 + .05 * .0499791693$$

$$\Delta X_{n+1} = .05 - \frac{-0.000021}{1.0 + .05 * .049979} = .050021$$

Let $\Delta X_n = .050021$; $\sin \Delta x - \Delta x$

$$= -0.0492; 1 + .00000873 = .951673$$

$$\Delta X_{n+1} = .05 + .051698 = .102$$

Thus,

$$\Delta x = .102$$

Rule 1.

Let

$$-1 \leq \cos \Delta x \leq 1$$

Dividing through by Δx ;

$$\frac{-1}{\Delta x} \leq \frac{\cos \Delta x}{\Delta x} \leq \frac{1}{\Delta x}$$

I know,

$$\pm \frac{1}{\Delta x} = 1$$

Substitution of the inverse decrement gives me,

$$-1 \leq \frac{\cos \Delta x}{\Delta x} \leq 1$$

Multiplying across by Δx ;

$$-\Delta x \leq \cos \Delta x \leq \Delta x$$

Thus,

$$\cos \Delta x = \pm \Delta x.$$

Rule no. 2.

We know,

$$\lim_{\Delta x \rightarrow 0} \frac{\sin \Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{d}{dx}(\cos \Delta x)}{1} = 1., \text{ because of L'hopita's Rule of Indeterminate forms.}$$

L'hopital's Rule of Indeterminate forms also suggests that

$$\lim_{\Delta x \rightarrow 0} \frac{\sin x}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(\Delta x)} = \frac{\lim_{\Delta x \rightarrow 0} \frac{d}{dx}(\sin x * \frac{\Delta x}{\Delta x})}{\frac{d}{dx}(\Delta x)}$$

$$= \frac{\Delta x^{-1} * \lim_{\Delta x \rightarrow 0} \frac{d}{dx}(\sin x * \Delta x)}{\lim_{\Delta x \rightarrow 0} \frac{d}{dx}(\Delta x)} = \lim_{\Delta x \rightarrow 0} \frac{\sin x * \frac{d}{dx}(\Delta x) + (\Delta x) * \cos x}{\frac{d}{dx}(\Delta x)}$$

$$= \lim_{\Delta x \rightarrow 0} \sin x = 0$$

Derivative of $\cos x$

Let $f(x) = \cos x$.

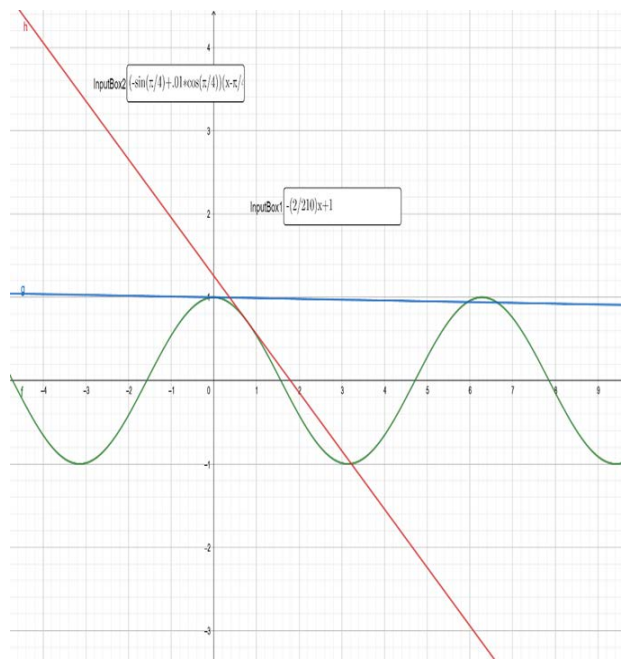
And its derivative of limits-

$$\frac{d}{dx}(\cos x) = \lim_{\Delta x \rightarrow 0} \left[\frac{\cos(x+\Delta x)}{\Delta x} - \frac{\cos(x)}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\cos x * \cos \Delta x}{\Delta x} - \frac{\sin x * \sin \Delta x}{\Delta x} - \frac{\cos x}{\Delta x} \right]$$

Substitution of the new values into the difference quotient gives me-

$$\frac{d}{dx}(\cos x) = \lim_{\Delta x \rightarrow 0} \left[\frac{\cos(x+\Delta x)}{\Delta x} - \frac{\cos(x)}{\Delta x} \right] = \Delta x^2 * \cos x - \sin x$$

Because $\Delta x^{-1} = 1$



Graph 1

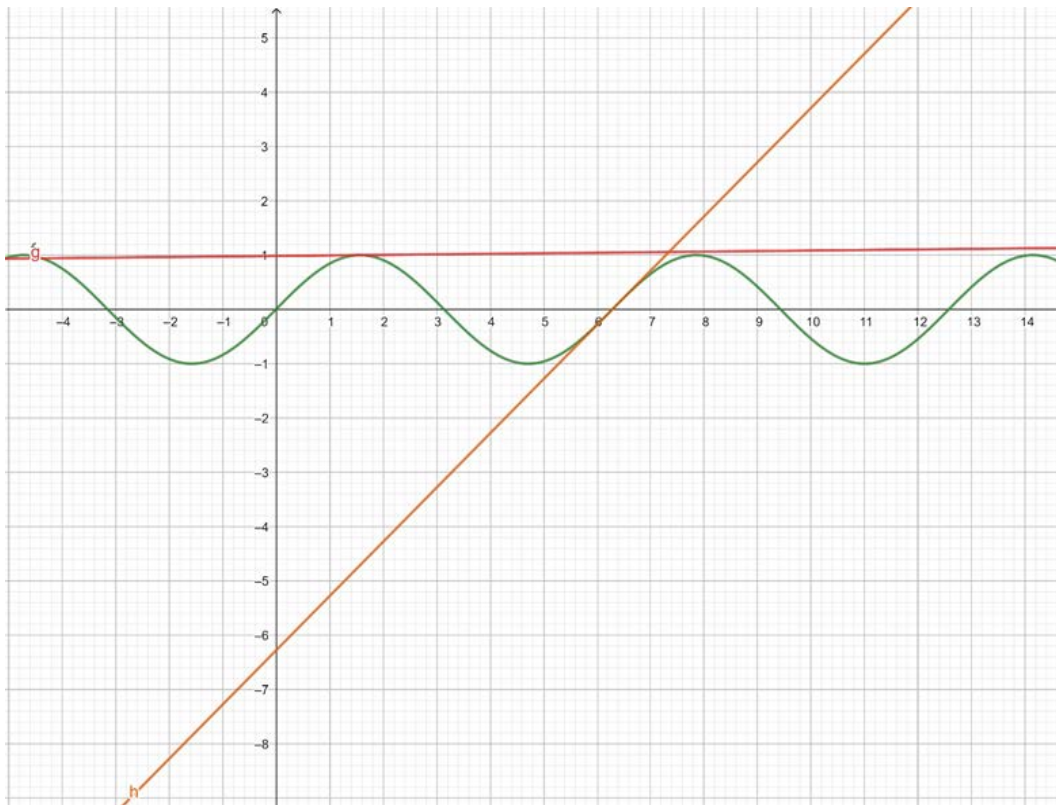
Derivative of $\sin x$

Therefore, we conclude with high degree of certainty that

$$\frac{d}{dx}(\sin(x)) = \lim_{\Delta x \rightarrow 0} \left[\frac{\sin x * \cos \Delta x}{\Delta x} + \frac{\cos x * \sin \Delta x}{\Delta x} - \frac{\sin x}{\Delta x} \right]$$

And substitution of these values into the difference quotient gives me-

$$\frac{d}{dx}(\sin(x)) = \lim_{\Delta x \rightarrow 0} \left[\frac{\sin(x+\Delta x)}{\Delta x} - \frac{\sin(x)}{\Delta x} \right] = \cos x + \Delta x^2 * \sin x$$



Graph 2

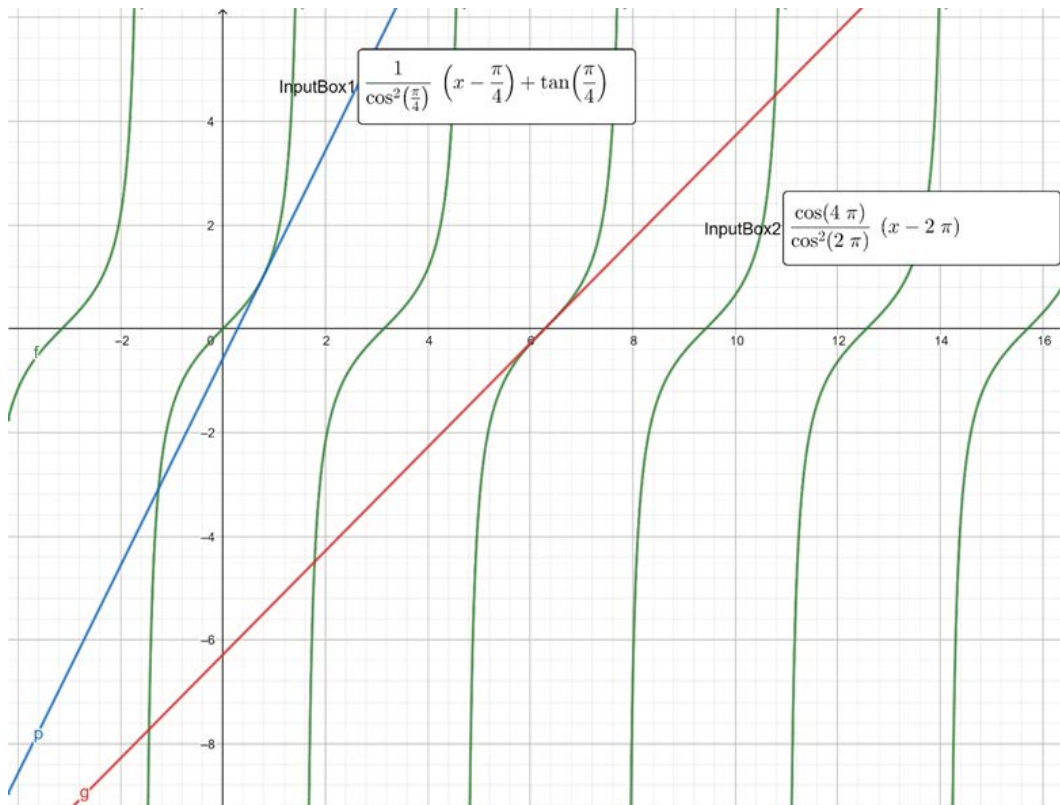
Derivative of $\tan x$

$$\frac{d}{dx}(\tan x) = \lim_{\Delta x \rightarrow 0} \left[\frac{\sin(x+\Delta x)}{\cos(x+\Delta x)} - \frac{\sin x}{\cos x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\frac{\sin x * \cos \Delta x + \cos x * \sin \Delta x}{\Delta x}}{\frac{\cos x * \cos \Delta x - \sin x * \sin \Delta x}{\Delta x}} - \frac{\frac{\sin x}{\Delta x}}{\frac{\cos x}{\Delta x}} \right]$$

$$\frac{d}{dx}(\tan x) = \lim_{\Delta x \rightarrow 0} \left[\frac{\frac{\cos x * \sin x * \cos \Delta x}{\Delta x} + \frac{\cos x^2 * \sin \Delta x}{\Delta x}}{\frac{\cos x^2}{\Delta x} - \frac{\cos x \sin x * \sin \Delta x}{\Delta x}} - \frac{\frac{\sin x * \cos x - \sin x * \sin x * \sin \Delta x}{\Delta x}}{\frac{\cos x^2}{\Delta x} - \frac{\cos x \sin x * \sin \Delta x}{\Delta x}} \right] =$$

$$\frac{d}{dx}(\tan x) = \lim_{\Delta x \rightarrow 0} \left[\frac{\frac{\cos^2 x * \sin \Delta x}{\Delta x} - \frac{\sin^2 x * \sin \Delta x}{\Delta x} + 2 * \sin x * \cos x}{\cos^2 x} \right]$$

$$\frac{d}{dx}(\tan x) = - \left[\frac{\cos 2x - \sin 2x}{\cos^2 x} \right]$$



Graph 3

Derivative of $\sin^{-1}x$

Let $y(x) = \sin^{-1}x$.

Differentiating left and right sides implicitly, I get-

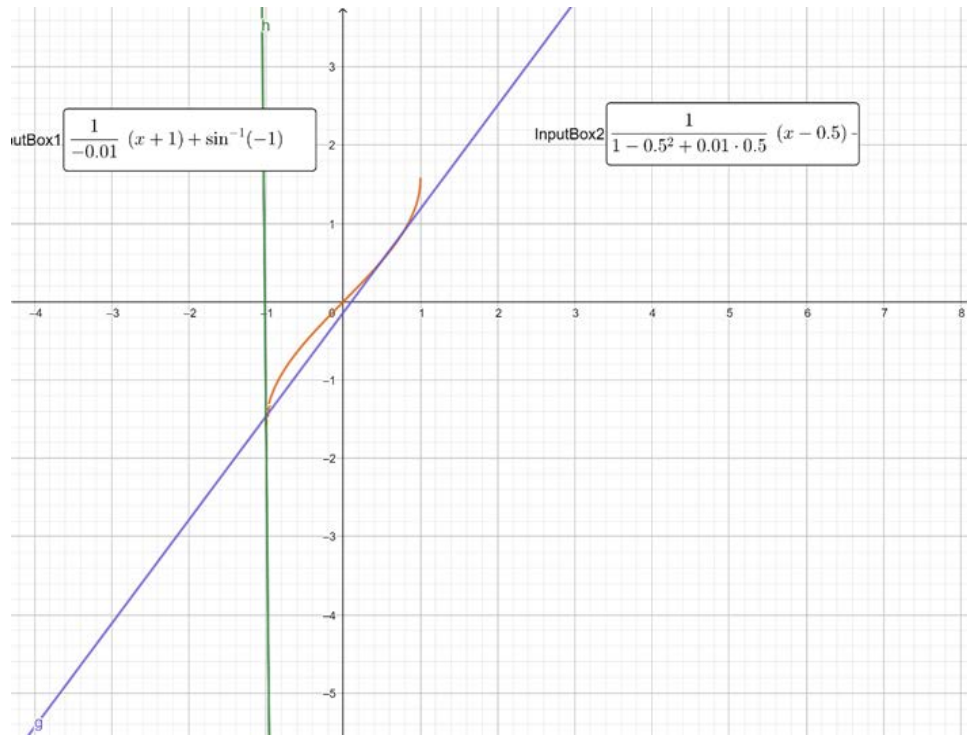
$$\sin y = x$$

$$(\cos y + \Delta y^2 * \sin y) * \frac{dy}{dx} = 1$$

thus

$$\frac{dy}{dx} = \frac{1}{(\cos y + \Delta y^2 * \sin y)}$$

$$\frac{dy}{dx} = \frac{1}{(\sqrt{1-x^2} + \Delta x^2 * x)}$$



Graph 4

The Derivative of the Natural Log Function.

Let $\xi(x) = \ln x$ with derivative defined as –

$$\lim_{n \rightarrow 0} \left[\ln \left(\frac{x+\Delta x}{\Delta x} \right) - \ln \left(\frac{x}{\Delta x} \right) \right] = \lim_{n \rightarrow 0} \left[\ln \left(\frac{x+\Delta x}{x} \right) \right]$$

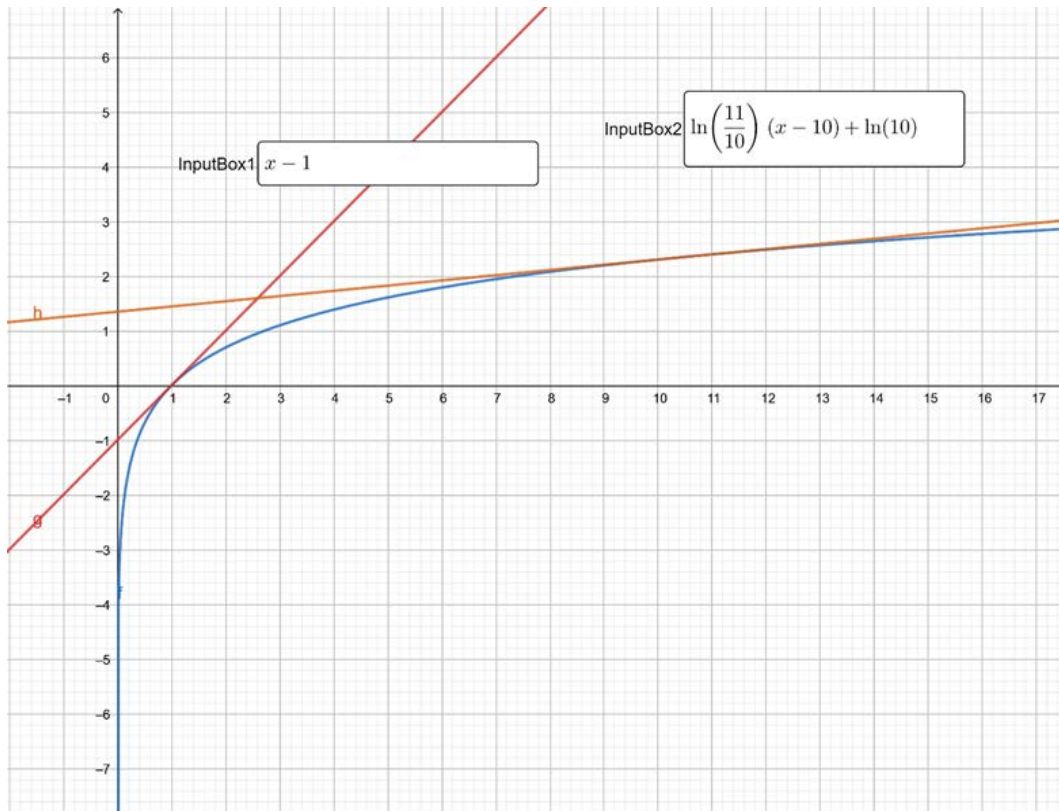
Extracting the decrement of the numerator gives me-

$$= \lim_{\Delta x \rightarrow 0} \left[\ln \left(\frac{\Delta x * (x * \Delta x^{-1} + 1)}{x} \right) \right] = \lim_{\Delta x \rightarrow 0} \left[\ln 1 - \ln 1 + \ln \Delta x + \ln \left(\frac{x+1}{x} \right) \right] = \ln \left(\frac{x+1}{x} \right)$$

I know $\ln 1 = 0$ and $-\ln 1 + \ln \Delta x = -(\ln 1 - \ln \Delta x) = -\ln \left(\frac{1}{\Delta x} \right) = -\ln (\Delta x^{-1}) = -\ln (1) = 0$.

Thus,

$$\frac{d}{dx} (\ln x) = \ln \left(\frac{x+1}{x} \right)$$



Graph 5

Example-

Let $X(x) = \sqrt{x}$.

To find the tangent line at the origin of the graph. Square both sides to get-

$$X^2(x) = x$$

Differentiating both sides, implicitly for $n=2$ at the left side between $0 \leq k \leq 2$

$$\frac{d}{dx} (X^2(x)) = \frac{2! * X^{2-0} * \Delta x^{0-1}}{0! * (2-0)!} + \frac{2! * X^{2-1} * \Delta x^{1-1}}{1! * (2-1)!} + \frac{2! * X^{2-2} * \Delta x^{2-1}}{2! * (2-2)!} - \frac{X^2}{\Delta x^1}; \text{ power rule}$$

Thus,

$$\frac{d}{dx} (X^2(x)) = (2x + \Delta x) * \frac{d(X(x))}{dx} = (2x + .1) * \frac{d(X(x))}{dx} = 1$$

Dividing by $(2x+.1)$ gives me-

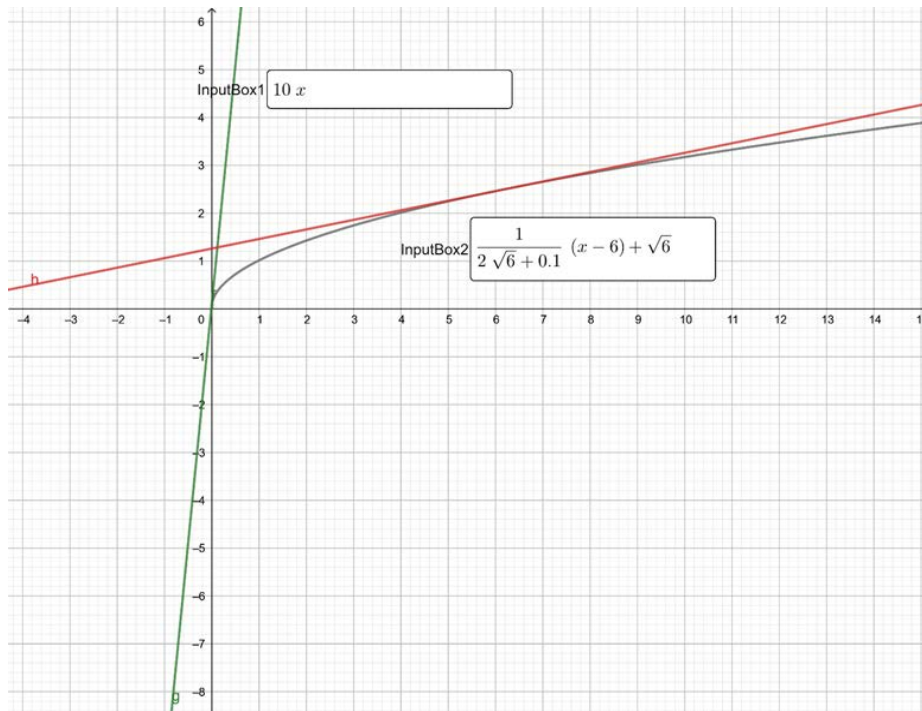
$$\frac{d(X(x))}{dx} = \frac{1}{2x+.1}$$

To compute the tangent line at the origin, we set $x=0$ and thus, the derivative at the origin becomes-

$$\frac{d(X(x))}{dx} = \frac{1}{.1} = 10; \text{ the rate of change at the origin of } \sqrt{x}.$$

The tangent line equation is expressed as of \sqrt{x}

$$T(x) = 10x$$



Graph 6

Newton’s Power Rule cannot explicitly compute the derivative of \sqrt{x} at its origin.

Accordingly, **Power Rule Theorem.**

See graph below.

Example-

Let $Y(x) = x^2$.

To find the tangent line at the origin of the graph I square both sides to get–

$$Y^2(x) = x^4$$

Differentiating both sides, implicitly for $n=2$; at the left side between $0 \leq k \leq 2$; the derivative consists of 4 terms; thus,

$$\frac{d}{dx}(Y^2(x)) = \frac{4! * X^{4-0} * \Delta x^{0-1}}{0! * (4-0)!} + \frac{4! * X^{4-1} * \Delta x^{1-1}}{1! * (4-1)!} + \frac{4! * X^{4-2} * \Delta x^{2-1}}{2! * (4-2)!} + \frac{4! * X^{4-3} * \Delta x^{3-1}}{3! * (4-3)!} + \frac{4! * X^{4-4} * \Delta x^{4-1}}{4! * (4-4)!} - \frac{X^4}{\Delta x^1};$$

power rule

The derivative of $Y(x)^2$ is $2Y+1$ and dividing the left term with the term on the right side gives me–

$$\frac{d}{dx}(Y(x)) = \frac{4*x^3+.6*x^2+.04*x+.001}{2*Y+.1}$$

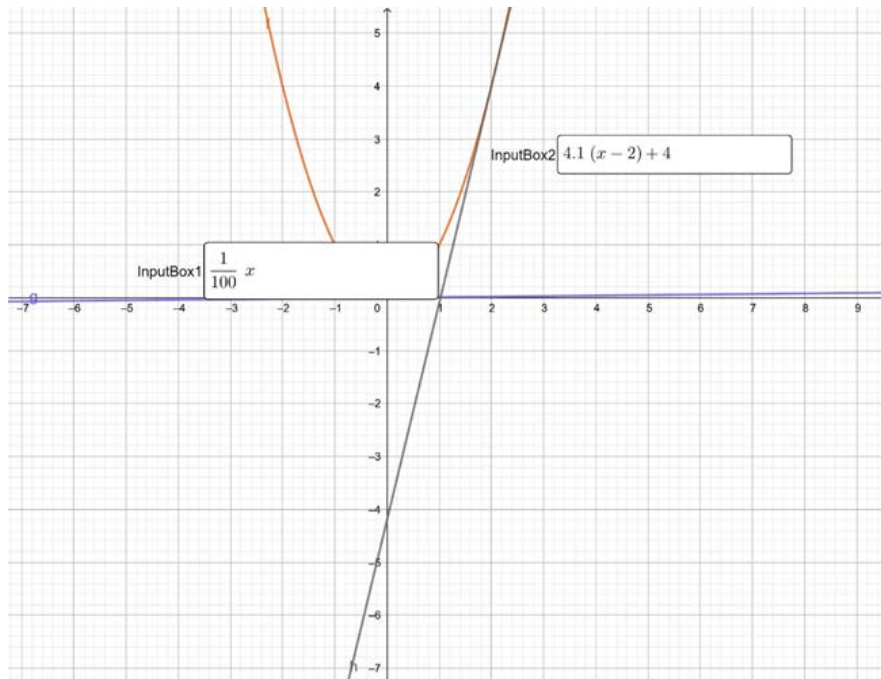
Therefore, the derivative of the parabola at the origin at $x=0; y=0; \frac{d}{dx}(Y(x)) = \frac{.001}{.1} = \frac{1}{100}$

The flat-tangent line equation with a rate of change of $\frac{1}{100}$ is expressed as –

$$T(x) = \frac{1}{100} x ; \text{ graph 2}$$

If $x= 2$; then, the rate of change at that instantaneous point of the curve is-

$$T(x) = \frac{34.481}{8.1}(x - 2) + 4$$



Graph 7

$$\text{Let } f(x) = x^{\frac{4}{3}}.$$

Cubing both sides, gives me

$$f(x)^3 = x^4$$

Differentiating implicitly both sides grant a derivative of y on the left side with an explicit derivative of x on the right side of the function.

That gives me

$$(3 * f(x)^2 + .3 * f(x) + .01) * \left(\frac{dy}{dx}\right) = 4 * x^3 + .6x^2 + .04 * x^1 + .001$$

The derivative $\frac{d}{dx} (x^{\frac{4}{3}}) = \frac{4*x^3+.6*x^2+.04*x^1+.001}{(3*f(x)^2+.3*f(x)+.01)}$; Power Rule Theorem.

The tangent line at the origin is equal to $\frac{d}{dx} (x_0^{\frac{4}{3}}) = \frac{.001}{.01} = .1 = \frac{1}{10}$

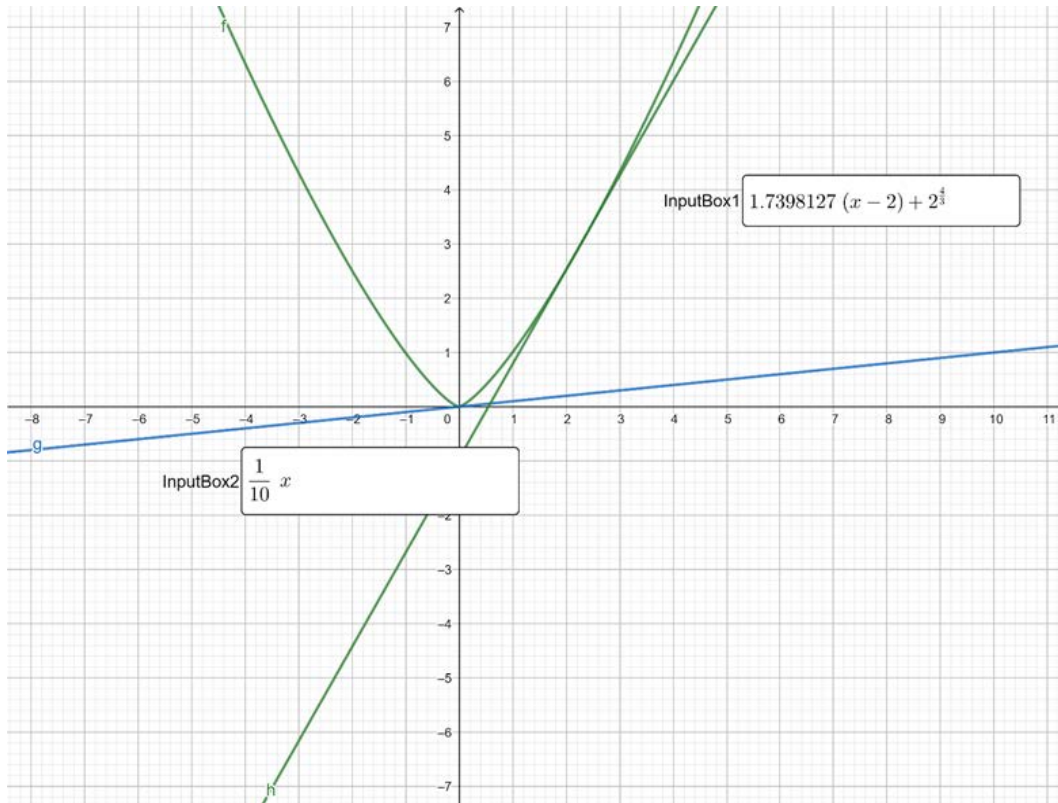
Tangent line equation of the irrational function is;

$$T(x) = \frac{1}{10} x$$

At $x = 2$, the tangent line is equal to $\frac{4*8+.6*4+.04*2+.001}{3*2^{\frac{8}{3}}+.3*2^{\frac{4}{3}}+.01} = \frac{34.481}{19.05+.7588+.01} = 1.7398127$

with tangent line equation

$$G(x) = 1.7398127(x - 2) + 2^{\frac{4}{3}}$$



Graph 8: See below for tangent lines of $x^{4/3}$.

The Derivative of the Circle

Let $x^2+y^2=16$.

Differentiating implicitly, the left and right sides, gives me-

$$(2x + .1) + (2y + .1) * \frac{dy}{dx} = 0$$

Dividing through with $(2x+.1)$ and $(2y+.1)$ gives me,

At $(0,4)$, the ratio of the perpendicular to horizontal equals $.1/8.1$ with tangent line

$$T(x) = \frac{.1}{8.1}(x) + 4 = \frac{1}{81}x + 4$$

and that's above the origin; 90 positive degrees from the positive horizontal axis.

At $(0,-4)$ the horizontal measurement equals $2*(-4)+.1=-7.9$ with a similar perpendicular measurement of 0, zero.

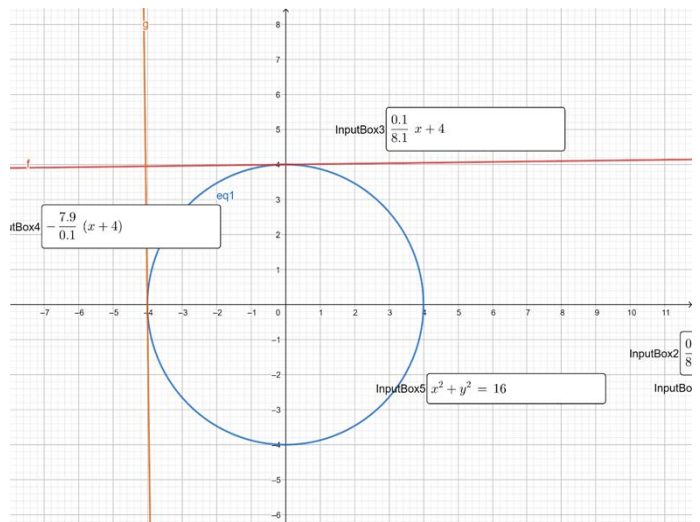
Thus, the tangent line equation is

$$-\frac{.1}{7.9}(x) - 4 = -\frac{1}{.79}x - 4$$

Now, let's compute the tangent line at the right and the left edge of the circle: $x=4$ and at $x=-4$.

At $x=4$, the tangent line equation is a vertical asymptote line equation of $T(x)=8.1/.1(x-4) = 81/1(x-4)$ and at $x=-4$, the tangent line equals 79 units of measurement with equation-

$$T(x) = -7.9/.1(x+4) = -79/1(x+4)$$



Graph 9

To explain the convergence of the tangent one to the asymptotic line, I assume the ratio of two sides of inverse tangent angle; that

$$\text{is if } \tan \varphi = -\frac{8.1}{.1} \rightarrow \text{ratio of } \frac{\text{perpendicular}}{\text{horizontal}}$$

To compute the angle φ , we compute the inverse tangent of the ratio of the two sides.

$$\tan^{-1}(\tan \varphi) = \varphi = \tan^{-1}\left(-\frac{8.1}{.1}\right) = |-1.56 \circ| = 1.56 \circ$$

Now, we compute the hypotenuse of our right triangle because of the perpendicularity of the vertical asymptote line:

$$\sqrt{(-8.1)^2 + (.1)^2} = 8.1006172$$

which is the measurement of the hypotenuse opposite to our perpendicular asymptote line which differs in measurement equal to .0006172. Thus, the perpendicular of our right triangle equal in measurement to the hypotenuse by less than .0006172 is equal to the vertical asymptote line with horizontal line equal to .1 units. Unit-wise the ratio of the two sides is- 1: .0125.

$$\text{At } x = 2, T(x) = \frac{4.1}{4*\sqrt{3}+1} (x - 2) + 2 * \sqrt{3} ;$$

and

$$\text{at } x=-2, T(x) = \frac{-3.9}{4*\sqrt{3}+1} (x + 2) - 2 * \sqrt{3};$$

the corresponding tangent lines are graphed on the subsequent graphs following the vertical asymptote lines of Newton's Circle.

Other Tangent Lines.

Find the horizontal tangent line equation of $\sqrt{x - 1}$ at (1,0)

Squaring both sides gives me-

$$y^2=x-1$$

Differentiating both sides gives me-

$$(2y + .1) * \frac{dy}{dx} = 1$$

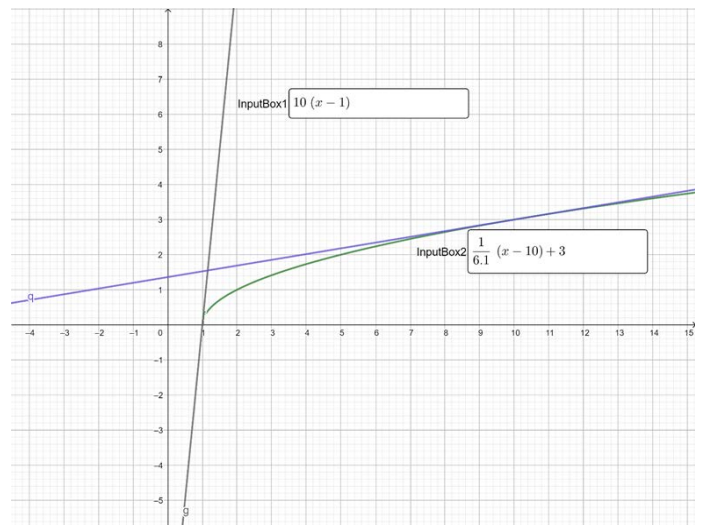
Dividing through by the term on the left, I get-

$$\frac{dy}{dx} = \frac{1}{2y+.1}$$

At the origin, (1,0), the tangent line equation is and becomes equal to-

$$10(x-1)$$

And if x=10, the tangent line equation, according to the above rule is and becomes equal to 1/6.1 (x-10) +3



Graph 10

Horizontal Tangent Lines

Method of solution of n-degree polynomial

Step 1. Square both sides.

Step 2. Implicitly differentiate the left and right side of the equation.

Step 3. Set the left side or the dependent side of the function equal to zero.

Step 4. Determine the solutions of the 2n-1 degree polynomial or the zeroes of the right side of the function of -x.

Step 5. Step 4 determines the minimums and maximums of the n-degree polynomial.

Step 6. Determine the logical derivative of the n-degree function.

Step 7. Plot tangent lines of the minimums and maximums given I step 5.

Horizontal Tangent Line of Irrational Functions with no horizontal intercept.

Given a function $I(x)=X^{m/n}$, its derivative is computed using the following procedure.

Step 1. Multiply the left and right sides by the lower degree of I(x); n.

Step 2. Implicitly differentiate I(x) on both sides with the Power Rule Theorem.

Step 3. Isolate dy/dx on the left side of the differential equation.

Step 4. Set x=0 and simplify the value of the quotient.

Step5. Plot tangent line at x=0.